Northgate High School Numeracy and Calculation Guidance Booklet

Introduction

To ensure consistency in teaching throughout Northgate High School. This Numeracy and Calculation guidance booklet has been produced to incorporate the importance of numeracy and mathematical skills across the curriculum. This handbook will give an overview of the different strategies used in our school to teach Mathematics throughout the Secondary curriculum.

Maths Curriculum

Our curriculum is designed to equip students to be able to use their knowledge of Mathematical concepts to solve unfamiliar problems and undertake complex reasoning. We use a **Mastery** approach that allows all students the opportunity to develop a deep understanding of Mathematical ideas with the focus on being able to answer Why as well as What and How.

Our students will learn conceptual understanding of mathematics using concrete material before using visual pictorial representation and then moving onto the abstract. (CPA)

We implement the mastery 5 big ideas:

1. Fluency = quick and efficient recall of facts and procedures
2. Variation = represent in more ways than one, sequencing the activities and paying attention to what stays the same and what changes
3. Mathematical thinking = reason and discussion (not merely passively received)
4. Representation = scaffold the work using concrete and pictorial representations, make connections
5. Coherence = ability to apply the concept to a range of contexts, lesson broken into small steps

Key stage 3 scheme of work is based on White Rose Maths https://whiterosemaths.com/.

Mastery Maths further reading: https://www.ncetm.org.uk/teaching-for-mastery/mastery-explained/five-big-ideas-in-teaching-for-mastery/

Teaching guidance

This document should provide information and guidelines to help produce consistency across the curriculum – it is not intended to be a prescription for teaching although some advice is given.

It recognises that not all students in a teaching group will have the same numerical skills and where unsure of an appropriate 'numerical level' teachers should consult with the Mathematics department.

It is recognised that there is never only one correct method. Students should be encouraged to develop their own correct methods, where appropriate, rather than being taught "set" ways. However, this document outlines many of the preferred methods that allow students to progress to harder skills and concepts.

All teachers should:

- Encourage students to show numerical working out and not just the answer
- Encourage the use of estimation particularly for checking work
- Encourage students to write mathematically correct statements

  example of a statement for calculating the area of a triangle

  incorrect statement: 30 x 5 = 150 ÷ 2 = 75
  correct statement: 30 x 5 = 150   150 ÷ 2 = 75

- Encourage students to ask themselves:

  'Can I do this in my head?'
  'Can I do this in using drawings or jottings?'
  'Do I need to use a written method?'
  'Do I need a calculator?'

Vocabulary

Consistency in the use of Mathematical language is important and allows the curriculum to be inclusive to all.

When referring to decimals use “three point one four” rather than “three point fourteen”.

It is important to use the correct Mathematical term for the type of average being used i.e mean, median or mode.

When using the equals sign, use the vocabulary ‘is the same as’ not ‘is the answer to’. Students need to have a clear understanding of what the equals sign means and can use it in a variety of different places within a calculation

When discussing each operation, vary the vocabulary used e.g. not always ‘plus’ for addition.

Jottings and Workings

‘In the teaching of English it is expected that children will be taught techniques for adapting their written and oral communication to suit their audience. If we extend this concept to the teaching of mathematics, it seems reasonable to accept that when children themselves are the sole audience for their work they should record in their own way, writing down those things that support them in their thinking; these are what we currently call *jottings*. With a different audience, for example, other children, teachers, or markers of exam questions, children need to be taught how to ensure that this audience can make sense of their recording; this is what we call *working*. For ‘supporting’ thinking we use jottings; for ‘explaining’ thinking we show our working.’

To jot or not to jot? by Ian Thompson
Mathematics in School 2004

Topics

This guide will provide guidance on the following topics.

1. Place value
2. Fact family
3. Addition
4. Subtraction
5. Multiplication
6. Division
7. Order of operation
8. Double and halving
9. Converting units of measure
10. Rounding and truncating
11. Fractions
12. Percentages
13. Ratio
14. Scale
15. Proportion
16. Inverse proportion
17. Pie Charts
18. Equations
19. Using formulae
20. Directed number

1. **Place value**

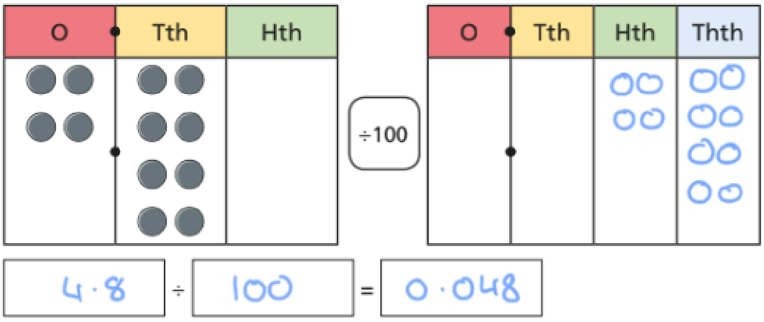
Vocabulary: place value, ones, tenths, hundredths, million

When writing larger numbers such as 3672 do not use a comma to separate the digits (3,672). A comma is often used throughout Europe as a decimal point and its use as a separator can confuse many of our EAL students.

Students are introduced to multiplying and dividing by powers of 10 in KS2. It is important the students have a conceptual understanding and not just a rule such as “add a zero”. They should understand that the digits move in the place value grid.
A few students may try and use formal methods for multiplying and dividing by powers of 10 such as column or short division. They should be encouraged to use the place value concept instead.

Multiplying and dividing by 0.1 can be a challenging concept for students. To avoid confusion multiplying by 0.1 is taught in year 7 and dividing by 0.1 in year 8.

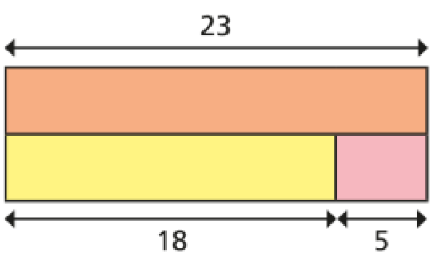
Place value grids can be used to support students.



2. **Fact family**

Vocabulary: fact family, inverse

Fact families are used to make connections between different facts for example addition and subtraction, multiplication and division. It helps them recognise equivalent forms and supports solving equations and rearranging formulae.



18 + 5 = 23   23 = 18 + 5

5 + 18 = 23   23 = 5 + 18

23 − 5 = 18   18 = 23 − 5

23 − 18 = 5   5 = 23 − 18

3. **Addition**

Vocabulary – Add, addition, total, sum, more than, plus, increase

All students will have been taught at primary school the traditional column method for addition. This is the quickest and most efficient method to use.

Add each column starting with the least significant number (right hand column). Ensure that children are still thinking of the value of the digits they are adding to check their answer is reasonable.

3587 + 672 = 4659

```
    3 5 8 7
+     6 7 2
    4 2 5 9
    1 1
```

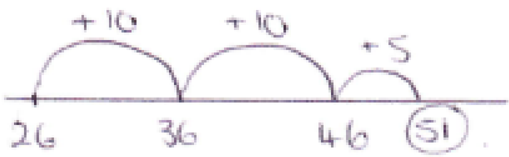
24.9 + 17.45 = 42.35

```
    2 4 . 9 0
+   1 7 . 4 5
    4 2 . 3 5
    1 1
```

Using a number line

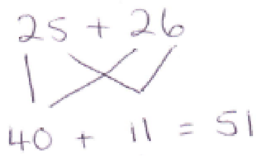
Teacher models jumps on numbered number lines to support understanding of the mental method.

26 + 25 = 51



Partitioning numbers

Here the students split the number using place value, tens and units.



4. **Subtraction**

Vocabulary – Subtract, subtraction, less than, difference, take away, decrease

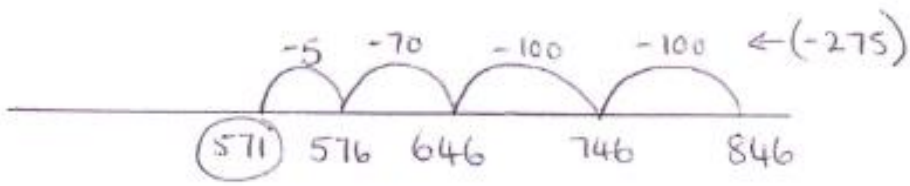
All students will have been taught at primary school the traditional column method for subtraction. This is the quickest and most efficient method to use.
Subtract each column starting with the least significant number (right hand column), subtract the bottom number from the top. If the number you are subtracting is larger and the result would be a negative then borrow from the next column.

$$\begin{array}{r} {}^{7}\cancel{8}{}^{1}46 \\ -\ \ 275 \\ \hline 571 \\ \hline \end{array}$$

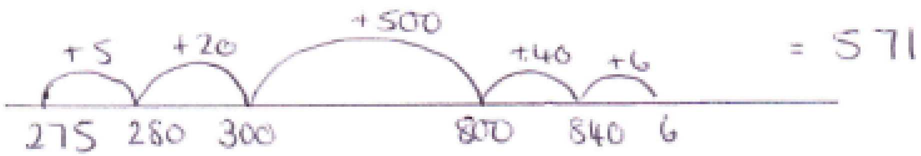
Using a number line

846 – 275 = 571

A number line can be used to count back



Or to count on
start at 275 and work out what you have to add on to get to 846
(this is a good mental method for working out change)



5. **Multiplication**

Vocabulary – Multiply, product, double, times
To be able to multiply students need to know their times tables and be able to recall multiplication facts to at least 10 x 10.

All students should be familiar with grid method/area model (method 1), column method (method 2) and repeat addition (method 3). Some may be familiar with the Napiers rod method/Chinese method (method 4).

Method 1

×	200	90	3
7	1,400	630	21

1400
630
+ 21
2051

Method 2

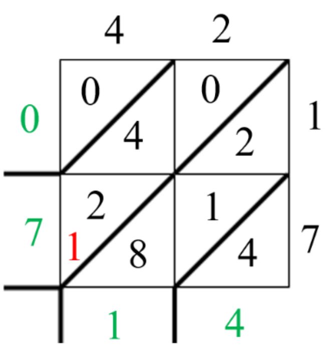
		2	9	3	
	×			7	
	2	0	5	1	
		6	2		

Method 3

		2	9	3	
		2	9	3	
		2	9	3	
		2	9	3	
		2	9	3	
		2	9	3	
	+	2	9	3	
	2	0	5	1	
		6	2		

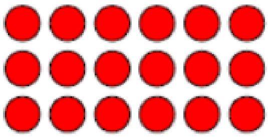
Method 4

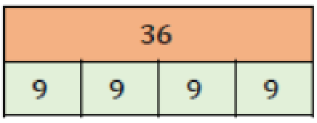
42 x 17



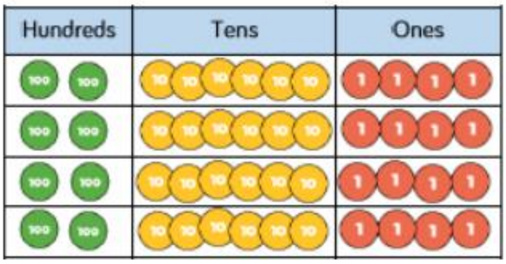
Useful Representations for multiplication

Students should have lots of experience in representing multiplication facts in arrays, exploring relationships between related facts (e.g. 5 x 3 and 3 x 5) and using the fact that multiplication is repeat addition.

Arrays:  6 x 3 = 18 = 6 + 6 + 6

Bar models: 

9 x 4 = 4 x 9 = 36

Place value: 

264 x 3 = 792

6. **Division**

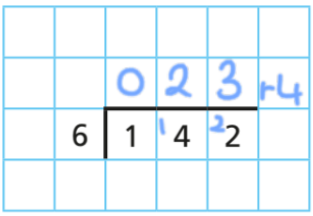
Vocabulary – Divide, division, remainder, quotient, equal groups of, half, inverse of multiplication

Students have studied both short division (method 1) and long division at KS2.

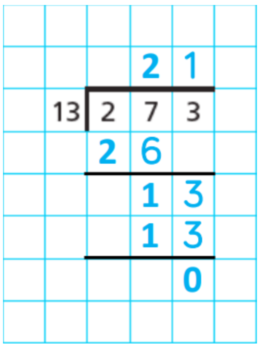
It is useful for all methods to encourage the students to write the times tables down to support them with the cognitive load.

It can be use to split a calculation into factors. E.g. 8808 ÷ 24 is the same as 8808 ÷6 ÷ 4

Method 1



Method 2



Method 3

Alternative method is to divide by chunking 42 ÷ 3 = 14

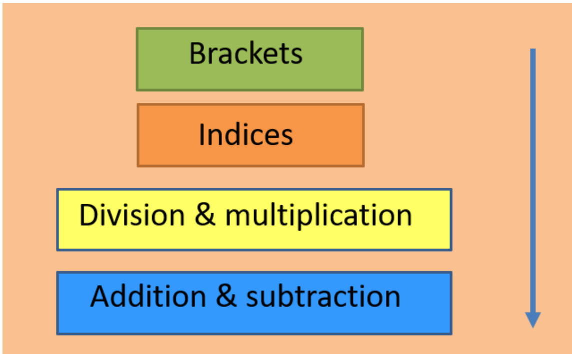
10 x 3 = 30

4 x 3 = 12

7. **Order of operation**

Vocabulary: order, operation, priority

Most students are familiar with the mnemonic BIDMAS to help them remember the order to perform operations.
A common misconception is that division must happen before multiplication, however in the situation when there are both operators then the order is from left to right.



Example:

a) $4 + \underline{3 \times 5} = \boxed{19}$ b) $\underline{12 \div 4} + 2 = \boxed{5}$ c) $7 + \underline{3^2} \times 2 = \boxed{25}$

8. **Doubling and halving**

Vocabulary: double, half, 50%

Halving numbers is a useful skill for students to acquire as it will support them with fractions and percentages. Dividing by 4 can be taught by halving and halving again.

They can use formal column method for multiplying and dividing by 2.

It should also be encouraged to use a partition method to multiply and divide by 2.

Example:

Half of 364 = 182

Half of 300 = 150

Half of 60 = 30

Half of 4 = 2

150 + 30 + 2 = 182

9. **Converting units of measure**

Vocabulary: metric, milli-, centi-, kilo-, convert, litre, gram, metre

When students convert metric units they need to understand the different types of metric units – length, mass and capacity.
It is useful for students to understand units in context to real-life, for example an adult male is approximately 1.8m in height.

Students need to understand the relative size of these different measures to help them understand the connection between them. This will help them see whether they need to multiply or divide, rather than relying on just remembering;

for example 1Kg is 1000 times as big as 1g.

$$1\text{kg} \xrightarrow{\times 1000} 1000\text{g}$$

$$3.2 \text{ kg} \xrightarrow{\times 1000} \boxed{3200} \text{ g}$$

Bar models can be used to represent conversions:

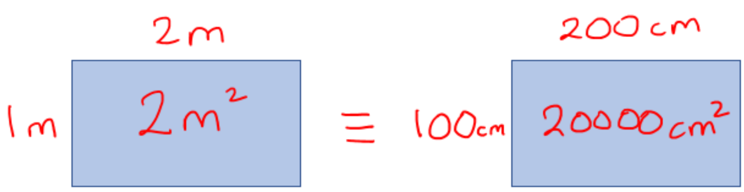
1 km	1 km	1 km	0.5 km
1,000 m	1,000m	1,000m	500m

3.5 km = $\boxed{3,500}$ m

A few students will be secure in converting areas in square centimetres to square metres.

A visual representation can be useful to support students.

For example: $2m^2 = 200 \times 100 = 20\ 000\ cm^2$



10. **Rounding and truncating**

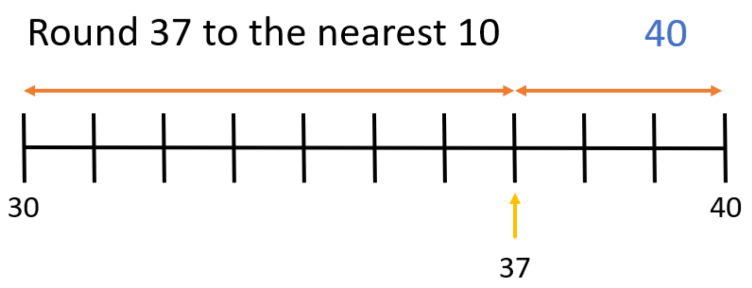
Vocabulary: round, approximate, nearest, halfway, bound, truncation

In key stage 2 students should be secure in rounding to the nearest 10, 100 and 1000. This is extended in key stage 3 to nearest decimal place and nearest significant figure.

It is important that students hear the language "round to the nearest" rather than "rounding up" and "rounding down" as this can lead to errors. The convention is that when there is a 5 in the relevant place value column, despite being exactly halfway between the two multiples, we round to the next one.

Number lines are a useful visual representation to support students with rounding.

Example:



This is further extended in key stage 4 to include error intervals and bounds. A value is given to a nearest position e.g. a length, l, is 3.6cm measured to the nearest cm. The lower bound is the smallest value it could be and the upper bound is the largest value it could be, this is often written as an error interval.
in this example $3.55cm \leq l < 3.65cm$

When we **truncate** a number, we find an estimate for the number without doing any rounding. To truncate a number, we miss off digits past a certain point in the number, filling-in zeros if necessary to make the truncated number approximately the same size as the original number.

To truncate a number to 1 decimal place, miss off all the digits after the first decimal place.

Examples

Truncate 3.784 to 1 decimal place and then to 2 decimal places.

3.7|84 truncated to 1 decimal place is 3.7

3.78|4 truncated to 2 decimal places is 3.78

Truncate 63,854 and 0.04988 to 3 significant figures.

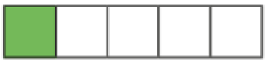
63,8|54 truncated to 3 significant figures is 63800

0.0498|8 truncated to 3 significant figures is 0.0498

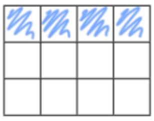
11. **Fractions**

Vocabulary: denominator, numerator

Students should be presented with, and be expected to represent, fractions in many ways to ensure conceptual understanding of what a fraction is.

Bar model (1/5) 

Number line (3/4) 

Equal parts  (4/12 = 1/3)

To Add and subtract the students should be encouraged to make the denominator the same.

$$\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$$

$$\downarrow \qquad \downarrow$$

$$\frac{5}{10} + \frac{4}{10}$$

To multiply fractions the student needs to multiply the numerators and multiply the denominators.

$$\frac{1}{2} \times \frac{2}{5} = \frac{2}{10} = \frac{1}{5}$$

To divide fractions students may use a simple mnemonic KFC (Keep Flip Change)

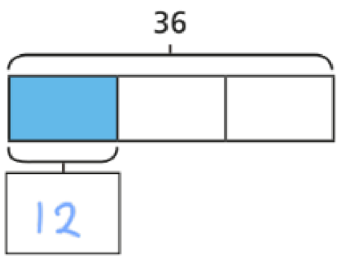
$$\frac{1}{2} \div \frac{2}{5}$$

$$K \downarrow \quad \downarrow C \quad \downarrow F$$

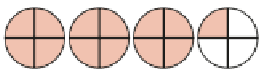
$$\frac{1}{2} \times \frac{5}{2} = \frac{5}{4}$$

A bar model is a useful representation to support students with calculating fractions of an amount. It helps to visualise that you divide by 3 to find one third of an amount.

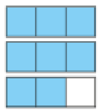
$\frac{1}{3}$ of 36 = $\boxed{12}$



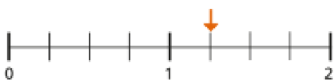
Visual representation can help to convert between mixed and improper fractions. Once secure with the pictures they can move on to numbers only and then the more abstract approach of whole number multiplied by denominator and then add the numerator.



$\boxed{3\frac{1}{4}}\ \boxed{\frac{13}{4}}$



$\boxed{2\frac{2}{3}}\ \boxed{\frac{8}{3}}$



$\boxed{1\frac{1}{4}}\ \boxed{\frac{5}{4}}$

$$2\frac{1}{3} = \underset{\text{whole}}{\frac{3}{3}} + \underset{\text{whole}}{\frac{3}{3}} + \frac{1}{3} = \frac{7}{3}$$

$$2\frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3}$$

A **calculator** can be used to convert fractions to decimals to percentages.
The recommended calculator for school use is the Casio FX83GTX. The students can enter the fraction using the fraction button and then press the  button to convert it to the decimal form or the format button .

12. **Percentages**

Vocabulary: percent, equivalent, of

Students will use a "build up" method for finding percentages of amounts and the higher tier will use a multiplier. It is therefore useful to be familiar with both when teaching percentages.

Build up method:
Students often start with multiples of 10% and 5%

find 35% of 640 = 224

100% = 640
10% = 64
5% = 32
30% = 64 + 64 + 64 = 192
35% = 192 + 32 = 224

Multiplier method:
a) Find 35% of 640 = 0.35 x 640 = 224
This uses the fact that 35% is equivalent to 0.35

b) increase 640 by 35% = 1.35 x 640 = 864
This uses the fact that 100% + 35% = 135% and this is equivalent to 1.35

c) decrease 640 by 35% = 0.65 x 640 = 416
This uses the fact that 100% - 35% = 65% and this is equivalent to 0.65

The percentage multiplier method is important for calculating repeat percentage change as it is the most efficient method.

example:

A child's ball loses 15% of its energy every time it rebounds. By considering that the ball's kinetic energy is proportional to the height from which it was dropped, determine the height, to the nearest centimeter, that the ball must be dropped from so that it rebounds to 20cm on the fifth bounce.

Each bounce:
$100\% - 15\% = 85\%$
$85\% = 0.85$

Bounce 1 : $0.85 \times h$

Bounce 2 : $0.85 \times 0.85 \times h = 0.85^2 \times h$

⋮

Bounce 5 : $0.85^5 \times h = 20$

$h = 20 \div 0.85^5 = 45.07\ldots$

Check:
0.85×45
$0.85^5 \times 45 = 19.96\ldots$

$\boxed{45\text{cm}}$

Calculating Percentage Change:

A formula can be used to find the percentage change, this is a useful technique for all students to be familiar with.

$$\% \text{ change} = 100 \times \frac{(\text{final - initial})}{|\text{initial}|}$$

Some students may prefer to use the build-up method to find the percentage change.

Example:

Original area of countryside (Hectares) = 260

Remaining area of country side (Hectares) = 221

Build-up

Difference = $260 - 221 = 39$

$100\% = 260$

$\left.\begin{array}{l} 10\% = 26 \\ 5\% = 13 \end{array}\right\} 15\% = 39$

15% decrease

Formula.

$$\frac{221 - 260}{260} \times 100 = -15\%$$

13. **Ratio**

Vocabulary: Ratio, equal parts, simplest form, for every, scale factor
Students should be able to represent ratios pictorially.

Example representations:

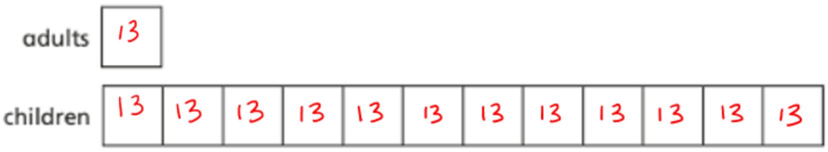
For every 3 blue there are 4 greens
blue : green = 3 : 4

Equivalent ratio

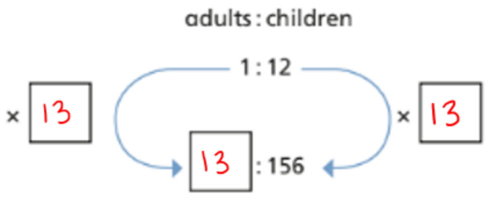
Equivalent ratio can be found by drawing the ratio or by multiplying.

1:12 is equivalent to 13:156

Method 1



Method 2



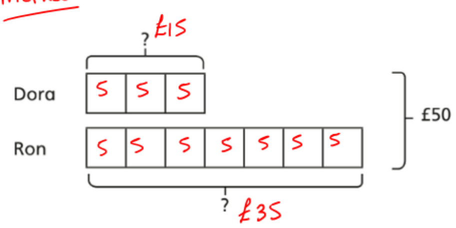
For method 2 it is important the students line up the numbers beneath the correct part.

Sharing in a given ratio

Picture methods or multiplication can be used.

Dora and Ron share £50 in the ratio 3 : 7

Method 1



50÷10 = 5

Method 2 50÷10 = 5

Dora : Ron

5 5 5 : 5 5 5 5 5

15 : 35

Method 3

50÷10 = 5

D : R

3 : 7

×5 15 : 35 ×5

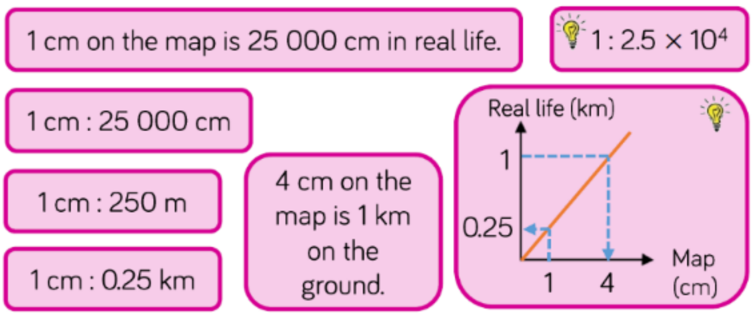
14. **Scale**

Vocabulary: scale factor, enlargement, object, image, length

The link between scale and ratio needs to be made explicit.

Bar models, circles, and lining up ratio are all valid methods that can also be applied to scale.

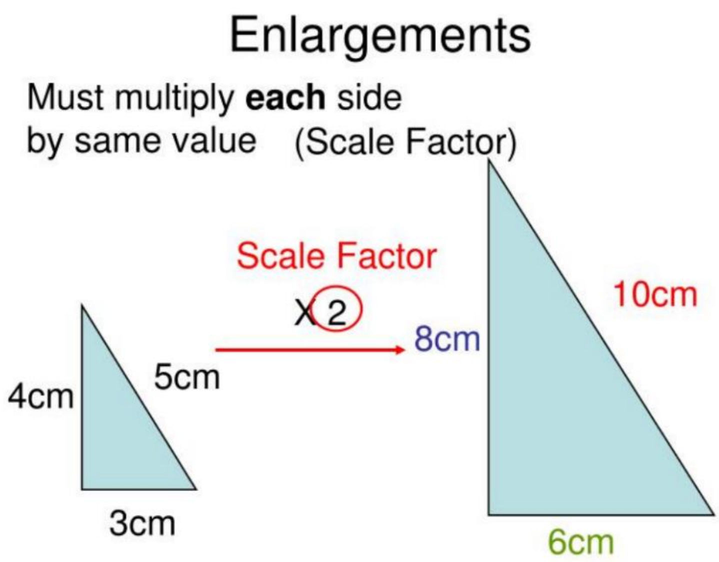
Example   Scale of 1:25 000



1cm = 250 m

×4 4cm = 1000m ×4

×2 8 cm = 2000 m ×2



15. **Proportion**

Vocabulary: Proportion, ratio, linear, variable, double, triple, scale factor

Students are familiar with proportion and should be able to think of examples in real-life where one variable doubles so does the other. The most common example is the price of items, if 3 bananas cost 75pence then 6 bananas cost £1.50

Proportion can be represented using bar models, proportion tables, build up methods and a formula.

Calculating speed. Formula speed = distance ÷ time

Method 1: Bar model

325 miles in 5 hours, what is the speed?

Method 2: Multiplying

325 miles in 5 hours, what is the speed?

Speed = miles in one hour

distance	time
325	5
65	1

÷ 5 ÷ 5

65 mph.

Method 3: Ratio table

Speed 1.5 km in 50 seconds

× 30 = speed.

Time	Distance
50 seconds	1.5 km
50 seconds	1500 m
100 seconds	3000 m
5 seconds	150 m
1 second	30 m

÷ 30

Ratio tables

Ratio tables can be used to find other values.
They use multiplication to go from one variable to another.
They can use sequences to extend and find other values. They can double and triple to find other values in the table. They can *build* to find other values in the table (3N = 1N + 2N)

Example: Force = **mass** x acceleration

Force = 2N and acceleration of $0.4m/s^2$

÷2 1+2

×5

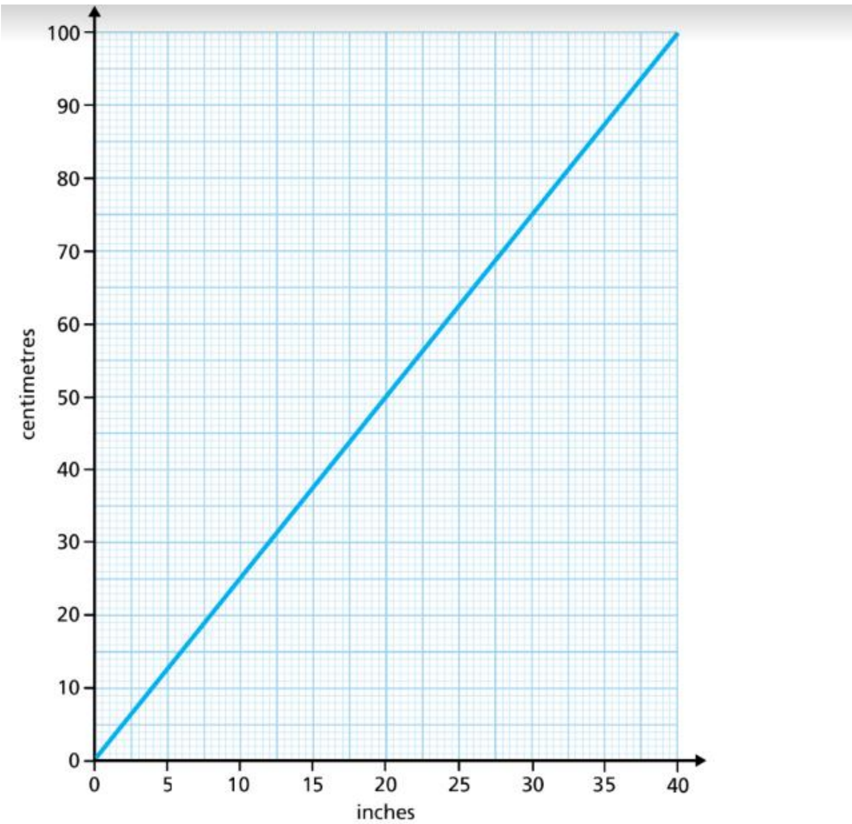
F (N)	1.0	2.0	3.0	4.0	5.0	6.0
a (m/s²)	0.2	0.4	0.6	0.8	1.0	1.2

×5 = mass

÷2 0·2+0·4

Proportion graphs

A graph is useful representation of proportion.
Plotting (0,0) and (10,25) the students can draw a linear graph and use this to convert further measurements.



16. **Inverse proportion**

Vocabulary: proportion tables

For inverse proportion when one variable is multiplied by a scale factor the other needs to be divided by the same scale factor.

It helps if the students think about what they expect to happen before they answer the questions. They should think is it going to get larger or smaller? This will prevent them from using direct proportion instead.

Example:

It takes 3 workers 9 hours to build a wall.

How long will it take one person? Is it longer or shorter?

Workers	Hours
3	9
1	27

$\div 3$ $\quad$ $\times 3$

1 worker = 27 hours

Time = 27 ÷ workers

It is useful to represent inverse proportion in a ratio table

T is inversely proportional to g. When $T = 10$ then $g = 2$
Using this information to complete the table of values for the relationship between T and g.

$\div 2$ $\quad$ $\div 5$ $\quad$ $\times 10$ $\quad$ $\times 2$

T	1	2	10	100	200
g	20	10	2	0·2	0·1

$\times 2$ $\quad$ $\times 5$ $\quad$ $\div 10$ $\quad$ $\div 2$

Form an equation that links T and g.

$$T = \frac{20}{g}$$

17. **Pie charts**

Vocabulary: pie chart, sector, key, fraction, proportion, protractor

Students are first introduced to pie charts in year 7. Students should link fractions of amounts and ratio and proportion to their work on pie charts.

Interpreting pie charts using fractions

Fractions can be used to interpret pie charts. E.g. if the red sector is 120 degrees then the red sector represents $\frac{120}{360}$ of the data. Students can then simplify this fraction to $\frac{1}{3}$ and use this to calculate values (the fraction button on the calculator can be used to simplify the fraction).

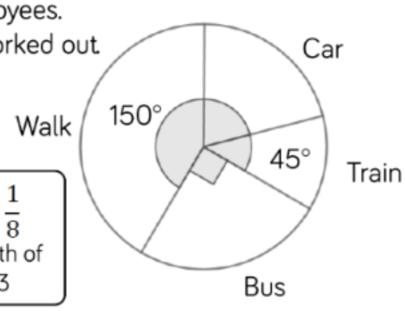
Example:

The pie chart shows the proportion of how a group of employees travel to work. There are 24 employees.
Explain how the following were worked out.

"a quarter of the people go by bus. So 6 people go by bus."

$\frac{150}{360} \times 24 = 10$
So 10 people walk.

$\frac{45}{360} = \frac{1}{8}$
One eighth of 24 is 3



Interpreting and drawing pie charts using ratio tables

A group of 45 people were asked to choose their favourite activity.

Complete the table and draw a pie chart to represent the data.

÷ 8

Result	Frequency	Angle of sector
read	17	136
draw	4	32°
play sport	9	72
play computer games	6	48°
other	9	72
Total	45	360

× 8

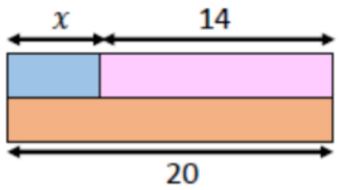
360 ÷ 45
= 8

18. **Equations**

Vocabulary: bar model, equality, variable, solve, inverse

If using x as a variable then it should be represented as x and not x.

One representation of equations is to use bar models and to link the work to fact families. This is a useful introduction to equation work.



Once they have understood the visual representation then they should be encouraged to move on the abstract. They should use a balancing method and inverses to help them solve equations.

Solve $2x - 3 = 30$

$+3 \qquad\qquad +3$

$2x = 33$

$\div 2 \qquad\qquad \div 2$

$x = 16.5$

19. **Using formulae**

Vocabulary: Formula, variable, substitution

Students will come into contact with formulas in Science and other curriculum subjects. They should be familiar with the Mathematical notation that is used such as

$2f$ means 2 multiplied by f

$\frac{a}{b} = a \div b$ however, is disciplines where a calculator is used then the student may wish to use their fraction button.

Before substituting into a formula it is useful for the students to write down the value of each variable. The next step is for them to write the formula with all the variables replaced.

Finally they can put the whole calculation into their calculators.

Example

$Kinetic\ energy = \frac{1}{2} \times mass \times velocity^2$

KE = ? Mass= 3kg Velocity = 16

KE $= \frac{1}{2} \times 3 \times 16^2$ = 384 Joules

If the question requires the students to work out a variable that is not currently the subject then a rearranging of the formula is required. This is a skill that the higher attaining students will be able to do but some students will find difficult. An alternative method is to substitute in the values and then solve.

$Kinetic\ energy = \frac{1}{2} \times mass \times velocity^2$

KE = 384J Mass= ? Velocity = 16

$384 = \frac{1}{2} \times m \times 16^2$

$384 = \frac{1}{2} \times m \times 256$ $\div\frac{1}{2}$

$\div\frac{1}{2}$ $768 = m \times 256$ $\div 256$

$\div 256$ $3 = m$

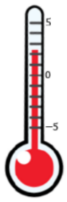
mass = 3kg

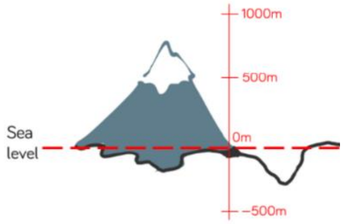
20. **Directed Number**

Vocabulary: positive, negative, sea level, number line, difference, zero pairs

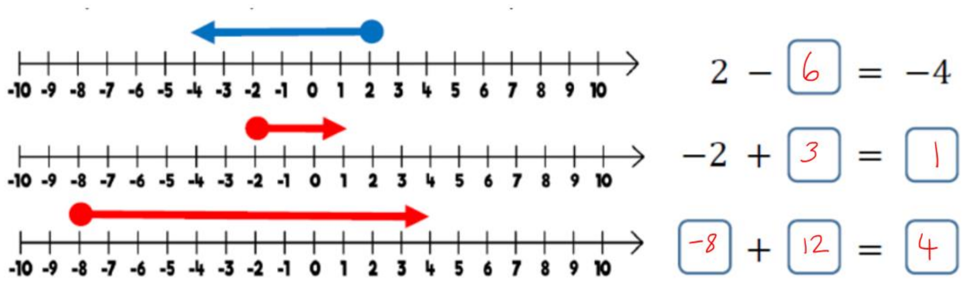
Students should recognise and use negative numbers in a variety of different representations, including real-life contexts and more abstractly with concrete manipulatives and written notation. Students should eb introduced to the reflective nature of positive and negative numbers on the number line e.g. knowing -4 and 4 are equidistant from 0. To avoid confusion “-4” should be read as “negative 4” and it can sometimes be written using brackets “(-4)”

Sea Level, thermometers and going up and down in lifts can help students put negative numbers in context.

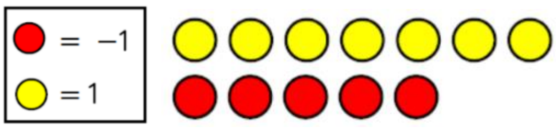




Number lines are a useful representation for directed number and can help students visualise adding and subtracting. Red arrows can be used to visualise adding and blue arrows for subtraction.

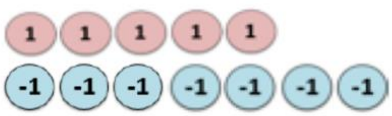


Concrete manipulatives and drawing circles can be used to support work with directed number. The concept of a zero pair (-1 and 1 = 0) can be used to calculate answers.



-5 + 7 = 2

If colours are not available then the student can write -1 and 1 in circles to support them.



5 + – 7 = -2